SHOE: Higher-Order Finite-Element Visualization

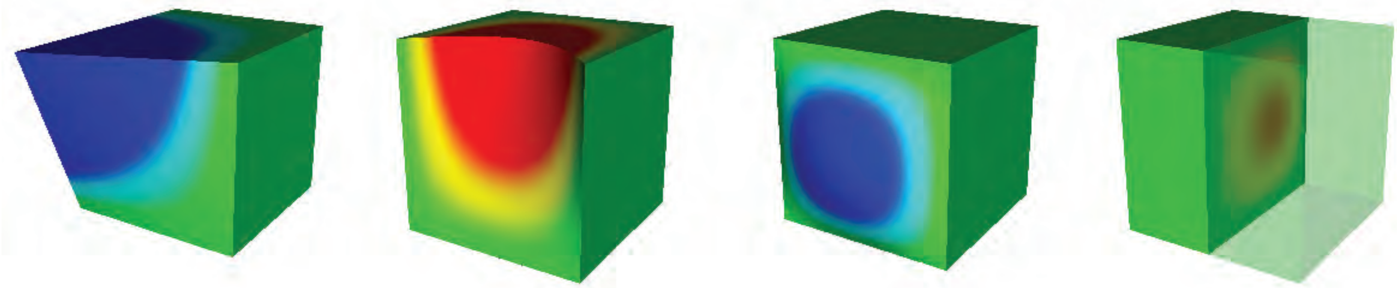

Figure 1. While low-order linear finite elements can only "stretch" at vertices (left), higher-order finite elements can have curvy edges, faces, and interiors (all four figures).

SHOE (Sandia Higher-Order Elements) is a research program to investigate the visualization of higher-order finite-element simulation results. Finite-element simulations typically use low-order (linear or occasionally quadratic) polynomials to approximate solutions of differential equations describing interesting situations. One example is solid mechanics, which approximates the deformation of a mechanical part in response to imposed forces. These finite-element simulations require the domain (i.e., the mechanical part) to be decomposed into many small regions in order for a low-order polynomial to approximate the true deformation of the part. By using higher-order polynomials, fewer elements may be needed. For certain types of simulations, reducing the number of high-order finite elements in a polynomial approximation can speed up a computer's calculation time by orders of magnitude.

However, since the polynomial approximation is simply a large set of numbers—the coefficients of the polynomials—understanding the results requires visualization. Unfortunately, not many visualization techniques can be applied to higher-order finite elements because existing techniques make assumptions about polynomials that do not hold for higher-order polynomials. This complicates the development and use of simulations that provide higher-order finite elements.

SHOE has been able to advance the state of the art in two ways:

- By developing a programming interface for higher order finite element visualization.
- By examining the mathematics of higher-order polynomials to develop robust analysis and visualization techniques.

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The programming interface that SHOE provides is now part of the Visualization Tool Kit (VTK) and is used by Simmetrix, Lehigh, and Sandia. This interface enables finite-element packages to describe the particular polynomials, known as the "polynomial basis," that they use to the visualization package without maintaining separate implementations of the polynomials for the finite element package and the visualization package. By using the information from the simulation packages, VTK can perform typical visualization operations by adaptively sampling the higher-order polynomials.

Sandia has also extended the programming interface to perform a more robust and correct sampling of higher-order finite elements when isosurfacing. To their detriment, current visualization techniques create a low-order approximation of the higher-order approximation of the differential equation. In contrast, SHOE simply divides the higher-order finite elements further into regions where the mathematical properties required for proper visualization still hold.

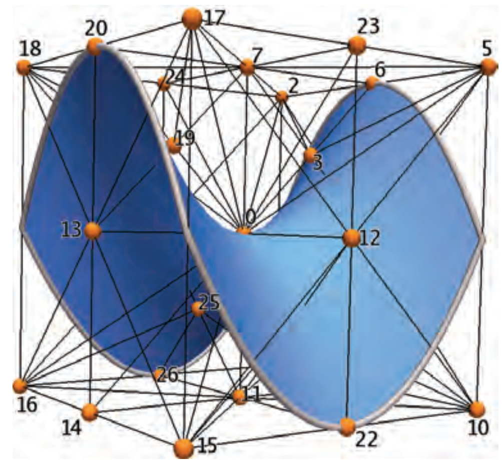

Figure 2. An isocontour produced by SHOE's contouring algorithm. The isocontour decomposes each finite element into regions guaranteed to produce the right shape and then adaptively triangulates the surface to the desired smoothness. Vertex 19 was inserted along the edge between vertices 0 and 20 so that a slightly higher isocontour (which would normally intersect edge 0–20 twice) would be properly handled.

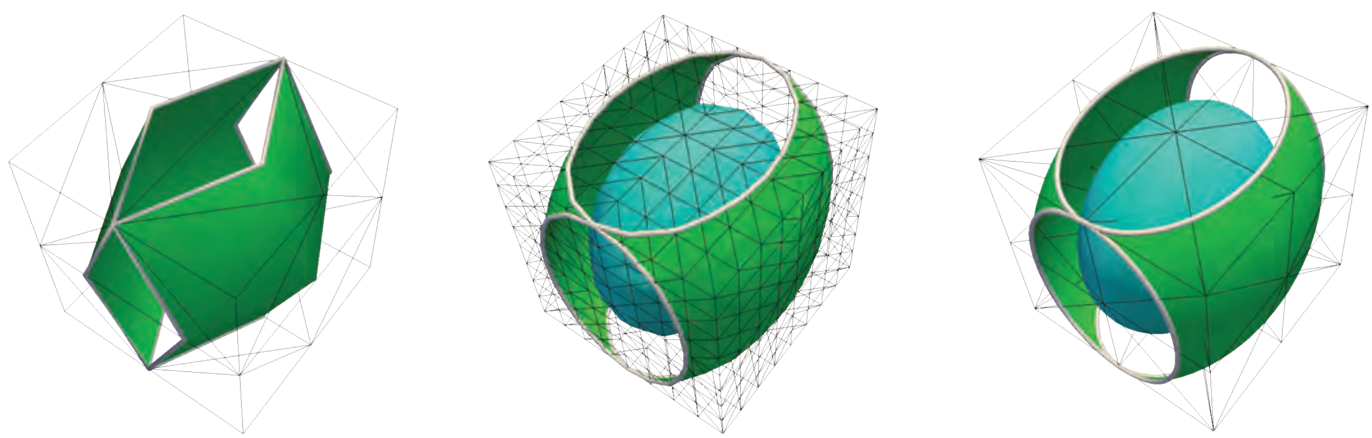

Figure 3. Sampling of higher-order polynomials (left) can miss features if the starting decomposition is too coarse. Brute-force sampling (middle) performs more work than is needed. SHOE's sampling provides the correct initial decomposition (right).

For more information, contact
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